

**ANEXO**

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**Tablas y Soluciones**

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### ALFABETO GRIEGO

A $\alpha$ : Alfa	H $\eta$ : Eta	N $\nu$ : Nu	T $\tau$ : Tau
B $\beta$ : Beta	$\Theta$ $\theta$ : Teta	$\Xi$ $\xi$ : Xi	$\Upsilon$ $\upsilon$ : Upsilon
$\Gamma$ $\gamma$ : Gamma	I $\iota$ : Iota	O $\omicron$ : Omicron	$\Phi$ $\phi$ : Fi
$\Delta$ $\delta$ : Delta	K $\kappa$ : Kappa	$\Pi$ $\pi$ : Pi	X $\chi$ : Chi
E $\epsilon$ : Épsilon	$\Lambda$ $\lambda$ : Lambda	P $\rho$ : Ro	$\Psi$ $\psi$ : Psi
Z $\zeta$ : Dzeta	M $\mu$ : Mu	$\Sigma$ $\sigma$ : Sigma	$\Omega$ $\omega$ : Omega

### LOGARITMOS

1.  $\log ab = \log a + \log b$        $\log \frac{a}{b} = \log a - \log b$
2.  $\log a^n = n \log a$                $\log \sqrt[n]{a} = \frac{1}{n} \log a$
3.  $\log 1 = 0$                            $\log_a a = 1$

## TRIGONOMETRIA

### Relación entre funciones trigonométricas

4.  $\cos u = \frac{1}{\sec u}$                        $\sec u = \frac{1}{\cos u}$
5.  $\sin u = \frac{1}{\csc u}$                        $\csc u = \frac{1}{\sin u}$
6.  $\tan u = \frac{\sin u}{\cos u}$                        $\cot u = \frac{\cos u}{\sin u}$
7.  $\sin^2 u + \cos^2 u = 1$
8.  $\sec^2 u = \tan^2 u + 1$
9.  $\csc^2 u = \cot^2 u + 1$

### Suma ángulos

10.  $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha$
11.  $\cos(\alpha - \beta) = \sin \alpha \cos \beta - \sin \beta \cos \alpha$
12.  $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$
13.  $\cos(\alpha + \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$
14.  $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$
15.  $\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$

### Ángulos dobles

16.  $\sin 2u = 2 \sin u \cos u$        $\sin u \cos u = 1/2 \sin 2u$
17.  $\cos 2u = \cos^2 u - \sin^2 u$   
 $\cos 2u = 2\cos^2 u - 1$        $\cos^2 u = 1/2 + 1/2 \cos 2u$   
 $\cos 2u = 1 - 2\sin^2 u$        $\sin^2 u = 1/2 - 1/2 \cos 2u$

$$18. \tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

### Ángulos medios

$$19. \sin \frac{u}{2} = \sqrt{\frac{1 - \cos u}{2}}$$

$$20. \cos \frac{u}{2} = \sqrt{\frac{1 + \cos u}{2}}$$

$$21. \tan \frac{u}{2} = \sqrt{\frac{1 - \cos u}{1 + \cos u}}$$

### Relaciones trigonométricas en función de sus ángulos medios

$$22. \sin u = 2 \sin \frac{u}{2} \cos \frac{u}{2}$$

$$23. \cos u = \cos^2 \frac{u}{2} - \sin^2 \frac{u}{2}$$

$$24. \tan u = \frac{2 \tan \frac{u}{2}}{1 - \tan^2 \frac{u}{2}}$$

### Transformación de sumas y diferencias de senos y cosenos

$$25. \sin x + \sin y = 2 \sin \frac{1}{2}(x + y) \cos \frac{1}{2}(x - y)$$

$$26. \sin x - \sin y = 2 \sin \frac{1}{2}(x - y) \cos \frac{1}{2}(x + y)$$

$$27. \cos x + \cos y = 2 \cos \frac{1}{2}(x + y) \cos \frac{1}{2}(x - y)$$

$$28. \cos x - \cos y = -2 \sin \frac{1}{2}(x + y) \sin \frac{1}{2}(x - y)$$

### Relación en un triángulo

$$29. \text{Ley seno: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$30. \text{Ley coseno: } a^2 = b^2 + c^2 - 2ab \cos A$$

## FORMULAS DE DERIVACIÓN

### Algunas de las formas más usadas

$$1. \frac{dc}{dx} = 0$$

$$2. \frac{dx}{dx} = 1$$

$$3. \frac{d}{dx}(u + v - w) = \frac{du}{dx} + \frac{dv}{dx} - \frac{dw}{dx}$$

$$4. \frac{d}{dx}(cu) = c \frac{du}{dx}$$

$$5. \frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$6. \frac{d}{dx}(u^n) = nu^{n-1} \frac{du}{dx}$$

$$7. \frac{d}{dx}(x^n) = nx^{n-1}$$

$$8. \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$9. \frac{d}{dx}\left(\frac{u}{c}\right) = \frac{du}{dx} \cdot \frac{1}{c}$$

$$10. \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}, \text{ siendo } y \text{ función de } u$$

$$11. \frac{d}{dx}(\ln u) = \frac{\frac{du}{dx}}{u} = \frac{1}{u} \frac{du}{dx}$$

$$12. \frac{d}{dx}(\log u) = \frac{\log e}{u} \frac{du}{dx}$$

$$13. \frac{d}{dx}(a^u) = a^u \ln a \frac{du}{dx}$$

$$14. \frac{d}{dx}(e^u) = e^u \frac{du}{dx}$$

$$15. \frac{d}{dx}(u^v) = vu^{v-1} \frac{du}{dx} + lu \cdot u^v \frac{dv}{dx}$$

## TABLAS DE INTEGRALES

### Integración por partes

Si  $u$  y  $v$  son funciones de la misma variable independiente, tenemos que:

$$\int u dv = uv - \int v du$$

### Algunas de las formas más usadas

$$16. \frac{d}{dx} \operatorname{sen} u = \cos u \frac{du}{dx}$$

$$17. \frac{d}{dx} \cos u = -\sin u \frac{du}{dx}$$

$$18. \frac{d}{dx} \tan u = \sec^2 u \frac{du}{dx}$$

$$19. \frac{d}{dx} \cot u = -\operatorname{csc}^2 u \frac{du}{dx}$$

$$20. \frac{d}{dx} \sec u = \sec u \tan u \frac{du}{dx}$$

$$21. \frac{d}{dx} \csc u = \csc u \operatorname{ctg} u \frac{du}{dx}$$

$$22. \frac{d}{dx} \operatorname{arc} \operatorname{sen} u = \frac{\frac{du}{dx}}{\sqrt{1-u^2}}$$

$$23. \frac{d}{dx} \operatorname{arc} \cos u = -\frac{\frac{du}{dx}}{\sqrt{1-u^2}}$$

$$24. \frac{d}{dx} \operatorname{arc} \tan u = \frac{\frac{du}{dx}}{1+u^2}$$

$$25. \frac{d}{dx} \operatorname{arc} \cot u = -\frac{\frac{du}{dx}}{1+u^2}$$

$$26. \frac{d}{dx} \operatorname{arc} \sec u = \frac{\frac{du}{dx}}{u\sqrt{u^2-1}}$$

$$27. \frac{d}{dx} \operatorname{arc} \csc u = -\frac{\frac{du}{dx}}{u\sqrt{u^2-1}}$$

$$1. \int f'(x) dx = f(x) + C$$

$$2. \int dx = x + C$$

$$3. \int a du = a \int du$$

$$4. \int u^n du = \frac{u^{n+1}}{n+1} + C$$

$$5. \int \frac{du}{u} = \ln u + C = \ln u + \ln C = \ln Cu$$

$$6. \int a^u du = \frac{a^u}{\ln a} + C$$

$$7. \int e^u du = e^u + C$$

$$8. \int \sin u du = -\cos u + C$$

$$9. \int \cos u du = \sin u + C$$

$$10. \int \sec^2 u du = \tan u + C$$

$$11. \int \csc^2 u du = -\cot u + C$$

$$12. \int \sec u \tan u du = \sec u + C$$

$$13. \int \csc u \cot u du = -\csc u + C$$

$$14. \int \tan u \, du = -\ln \cos u + C = \ln \sec u + C$$

$$15. \int \cot u \, du = \ln \sin u + C$$

$$16. \int \sec u \, du = \ln(\sec u + \tan u) + C$$

$$17. \int \csc u \, du = \ln(\csc u + \cot u) + C$$

$$18. \int \frac{du}{u^2 + a^2} = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$19. \int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \frac{u - a}{u + a} + C \quad u^2 > a^2$$

$$20. \int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \frac{a + u}{a - u} + C \quad u^2 < a^2$$

$$21. \int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$$

$$22. \int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln \left( u + \sqrt{u^2 \pm a^2} \right) + C$$

$$23. \int \sqrt{a^2 - u^2} \, du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \arcsin \frac{u}{a} + C$$

$$24. \int \sqrt{u^2 \pm a^2} \, du = \frac{u}{2} \sqrt{u^2 \pm a^2} \pm \frac{a^2}{2} \ln \left( u + \sqrt{u^2 \pm a^2} \right) + C$$

#### Formas racionales que contienen $a + bu$

$$25. \int (a + bu)^n \, du = \frac{(a + bu)^{n+1}}{b(n+1)} + C, \quad n \neq -1$$

$$26. \int \frac{du}{a + bu} = \frac{1}{b} \ln(a + bu) + C$$

$$27. \int \frac{udu}{a + bu} = \frac{1}{b^2} [a + bu - a \ln(a + bu)] + C$$

$$28. \int \frac{u^2 du}{a + bu} = \frac{1}{b^3} \left[ \frac{1}{2} (a + bu)^2 - 2a(a + bu) + a^2 \ln(a + bu) \right] + C$$

$$29. \int \frac{udu}{(a + bu)^2} = \frac{1}{b^2} \left[ \frac{a}{a + bu} + \ln(a + bu) \right] + C$$

$$30. \int \frac{u^2 du}{(a + bu)^2} = \frac{1}{b^3} \left[ a + bu - \frac{a^2}{a + bu} - 2a \ln(a + bu) \right] + C$$

$$31. \int \frac{udu}{(a + bu)^3} = \frac{1}{b^2} \left[ -\frac{1}{a + bu} + \frac{a}{2(a + bu)^2} \right] + C$$

$$32. \int \frac{du}{u(a + bu)} = -\frac{1}{a} \ln \left( \frac{a + bu}{u} \right) + C$$

$$33. \int \frac{du}{u^2(a + bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left( \frac{a + bu}{u} \right) + C$$

$$34. \int \frac{du}{u(a + bu)^2} = \frac{1}{a(a + bu)} - \frac{1}{a^2} \ln \left( \frac{a + bu}{u} \right) + C$$

#### Formas racionales que contienen $a^2 \pm b^2 u^2$

$$35. \int \frac{du}{a^2 + b^2 u^2} = \frac{1}{ab} \arctan \frac{bu}{a} + C$$

$$36. \int \frac{du}{a^2 - b^2 u^2} = \frac{1}{2ab} \ln \left( \frac{a + bu}{a - bu} \right) + C, \quad a^2 > b^2 u^2$$

$$37. \int \frac{du}{b^2 u^2 - a^2} = \frac{1}{2ab} \ln \left( \frac{bu - a}{bu + a} \right) + C, \quad a^2 < b^2 u^2$$

$$38. \int u(a^2 \pm b^2 u^2)^n \, du = \frac{(a^2 \pm b^2 u^2)^{n+1}}{\pm 2b^2(n+1)} + C$$

39.